# A brief summary of the Particle Finite Element Method

The following list of proposals are based on the Particle Finite Element Method (PFEM). This method allows to formulate fluid flow equations in a Lagrangian framework using flassical FEM and remeshing techniques (Idelsohn, Oñate, and Del Pin, 2004). Its formulation eases simulation of free surface flows and fluid-structure interactions, among others. In a simplified fashion, PFEM consists of: First, a discretization of the fluid using a cloud of particles, as shown in Fig. 1a. Second, a Delaunay triangulation to generate a mesh on the convex hull of the cloud (Fig. 1b). Third, the Alpha Shape algorithm (Edelsbrunner and Mücke, 1994) to remove those elements whose circumcircle radius exceeds a characteristic size of the mesh scaled by a $\alpha_{\text{shape}}$ parameter. In this way, depending on the value of $\alpha_{\text{shape}}$, large or highly distorted elements are removed from the triangulation, leaving a discretization of the fluid with a clear definition of its boundaries and free surface, as shown in Fig. 1c. Fourth, a FEM to discretize the Lagrangian Navier-Stokes equations, whose solution gives the nodal velocity (or displacement) and pressure (Fig. 1f). The new fluid position after one time step is obtained by the time integration scheme (Fig. 1e). Fifth, a check on the mesh quality to determine if re-meshing is necessary (Fig. 1d) or if the triangulation can be preserved for the next time step. A recent survey on PFEM and more details on the above steps can be found in (Cremonesi et al., 2020).

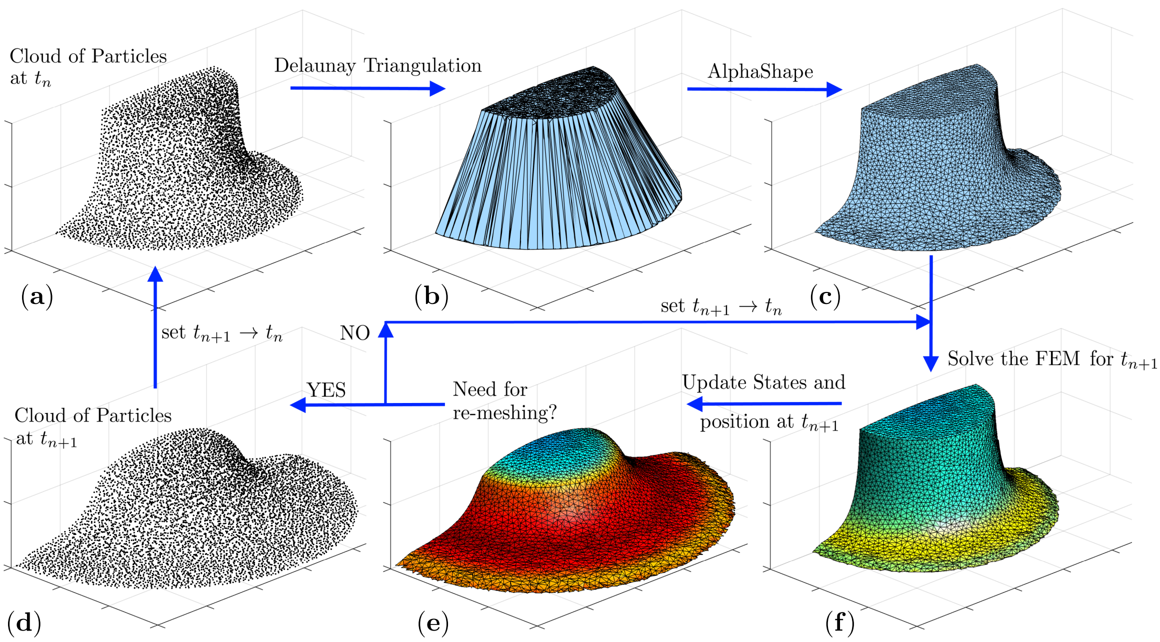


Figure 1: Illustration of the PFEM procedure using the collapse of a cylindrical water column problem. (a) Cloud of particles with known state variables at $t_n$. (b) Delaunay triangulation on the entire set of particles. (c) Large and distorted elements are removed from the triangulation using Alpha Shape algorithm (Edelsbrunner and Mücke, 1994). (f) Norm of nodal velocities for illustrating a FEM. (e) Illustration of updating state variables and nodal position. (d) The updated cloud of particles to be used in the next time step.

## References


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