# An extended analysis of time integration schemes in the PFEM for the simulation of fluid flows and fluid-structure interactions

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## 1 Introduction

The governing equations for body motion contain state variables and their time derivatives as unknowns. Commonly, nodal position, velocity and acceleration in solid mechanics, and nodal velocity and acceleration in fluid mechanics. The relationship between state variables and their time derivatives is usually unknown, and therefore must be approximated using time integration schemes. Several schemes have been proposed in the literature and their performances are well understood in computational mechanics. For example, Dettmer and Perić (2003) compare several time integration schemes in the finite element method for solving the incompressible Navier-Stokes equations in an Eulerian framework.

The LTAS-MN2L group develops a code based on the Particle Finite Element Method (PFEM) that solves the incompressible Navier-Stokes equations using a Lagrangian framework. The code includes several time integration schemes such as Backward Euler, Newmark and Generalized-$\alpha$. The latter also retrieves other time integration schemes depending on some combination of parameters. However, the implemented schemes in PFEM3D have not been extensively compared, so there are probably still observations and improvements to be made in this regard.

## 2 Objective

The objective of this master thesis is to compare extensively the different time integration schemes available in the PFEM3D code. The idea is to solve a set of benchmark problems in PFEM, such as the Dam Break (see Fig. 1), the water sloshing in a reservoir, and the fall of a water sphere in a reservoir (these can be found, for instance, in Franci and Cremonesi, 2017). In addition, the time integration schemes should be compared according to the list of examples in Dettmer and Perić (2003), with the purpose of replicating their results but now in a Lagrangian framework (the cited work uses an Eulerian framework). Ultimately, the analysis might include multiphysics problems involving fluid-structure interaction, thermo-fluids and phase changes.

**Background knowledge** for a smooth and successful development of the project: C++ (basic level) and Large deformation of solids (MECA0464-1)

## References

Cerquaglia, Marco-Lucio. 2019. *Development of a fully-partitioned PFEM-FEM approach for fluid-structure interaction problems characterized by free surfaces, large solid deformations, and strong added-mass effects*. Vol. PhD thesis, University of Liege. https://hdl.handle.net/2268/233166.

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Franci, Alessandro, and Massimiliano Cremonesi. 2017. "On the effect of standard PFEM remeshing on volume conservation in free-surface fluid flow problems." *Computational Particle Mechanics* 4 (3): 331–343.

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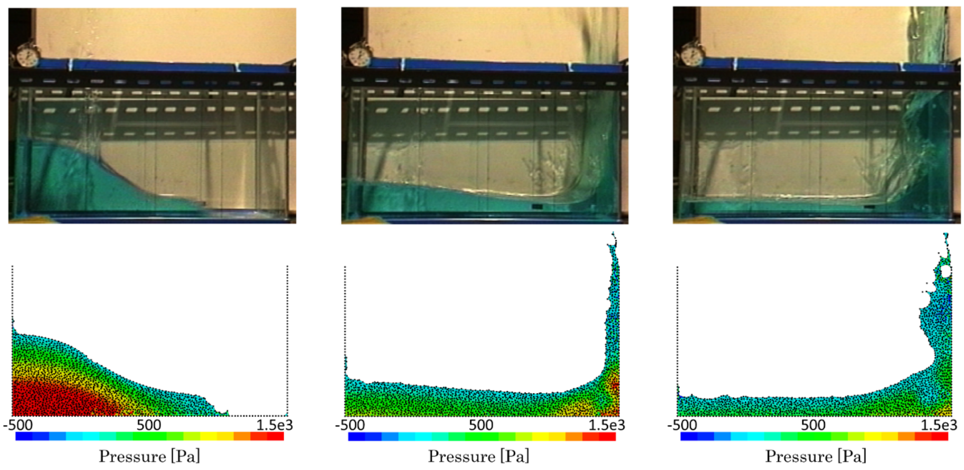


(a) $t = 0.2$s (b) $t = 0.4$s (c) $t = 0.6$s

Figure 1: Dam break problem solved using PFEM. Image from (Cerquaglia, 2019)