Node relocation optimization aiming at meshes with conformal interfaces

Eduardo Fernández, Simon Février, Martin Lacroix, Luc Papeleux,
Romain Boman, Jean-Philippe Ponthot

LTAS-MN2L, Aerospace and Mechanical Engineering Department, University of Liege, Belgium.

1 Introduction

The remeshing strategy of PFEM facilitates the detection and tracking of fluid boundaries when compared to Eulerian-based CFD methods, simplifying, for example, the simulation of free-surface fluid flows and fluid-structure interactions. However, compared to fixed-mesh methods, remeshing complicates efficient parallelization in distributed memory architectures, mainly because the mesh must be partitioned and sent to each process at each remeshing (see Fig. 1), demanding larger data communication as the mesh or the number of processes grow.

A strategy that has recently gained interest in the literature consists in reducing or avoiding remeshing in simulations with interfaces by adapting the position of nodes to the interface. The aim is to avoid changing the connectivity of the mesh to preserve the domain partition, but nevertheless achieving a conformal mesh, as shown in Fig. 2.

The node relocation methods found in the literature have been tailored to suit different needs. A group of methods are designed around interfaces being defined by a level set function (Schmidt et al., 2024; Quiriny et al., 2024; Chemin et al., 2025), and another group assumes that the interface is discretized by nodes and lines (in 2D), and surfaces (in 3D) (Garimella, Shashkov, and Knupp, 2004). Regardless of how the interface is represented, the idea of the node relocation methods is to solve an optimization problem that seeks to maximize the quality of the mesh or minimize some error function representative of the governing equations.

This master's thesis proposal is presented for the case in which the interface is discretized by points, lines and surfaces. The motivation is to pave the way towards a partitioned solver for fluid-structure interaction simulations and towards a body-fitted topology optimization method (Schmidt et al., 2024).

2 Objective

The aim of this work is to explore optimization algorithms for the relocation of nodes in order to obtain meshes conforming to a discretized interface. The goal of the work is to implement and analyze different optimization criteria that can be considered during node relocation, such as criteria based on

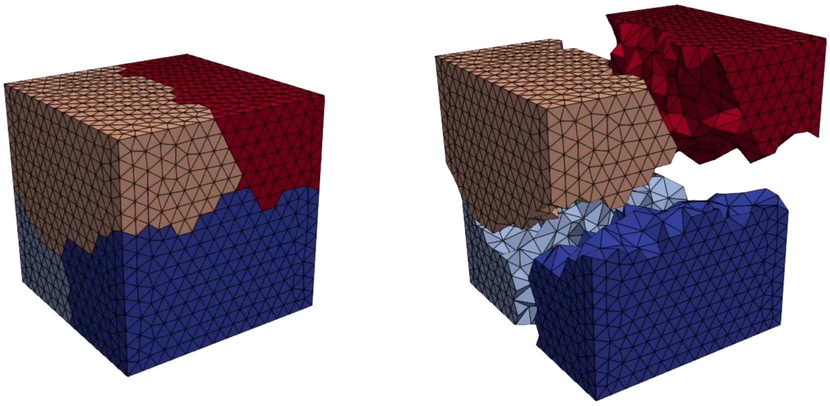

Figure 1: *Illustration of a domain decomposition in which the finite element mesh is split into 4 domains for distributed memory computation using 4 processes.*

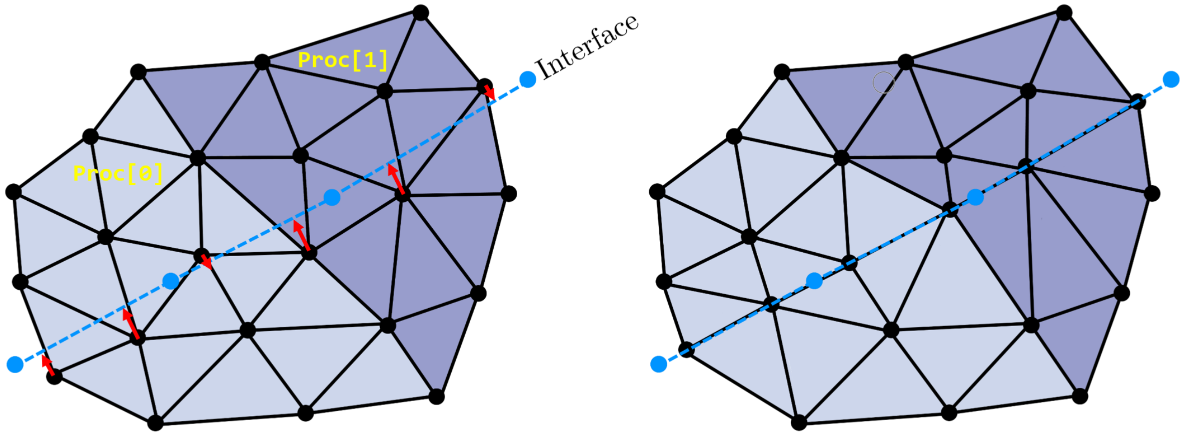

Figure 2: *On the left, a computational mesh distributed in 2 processes and an interface discretized by blue segments. On the right, the nodes close to the interface have been moved to accommodate the computational mesh to the interface.*

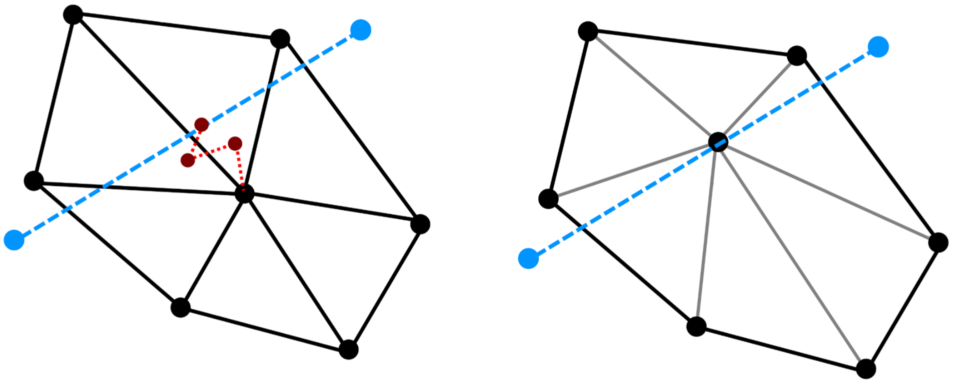

Figure 3: *Illustration of an optimization process that seeks to maximize the quality of the elements and minimize the distance between the node and the interface. On the left, iterations to optimize the position of a node, and on the right, the optimized new mesh.*

the Jacobian determinant of elements and/or on the distortion of the elements, considering in turn that the node must move towards and on the interface, in addition to defining criteria to determine which nodes will be relocated. To give an idea of the optimization problem, we can think of:

$$\begin{aligned} \max_{\mathbf{x}} \quad & \min(J) \\ \text{s.t.} \quad & \mathrm{dist}(\mathbf{x}, \mathrm{L}) = 0.0 \\ & Q \geq Q_{\mathrm{min}} \end{aligned} \tag{1}$$

where $\mathbf{x}$ represents the position of the nodes to be moved, $\min(J)$ the element with the smallest Jacobian determinant linked to the nodes that can be moved, $\mathrm{dist}(\mathbf{x}, \mathrm{L})$ the distance between the nodes to be moved and the interface, Q an element quality criterion, and Q_{min} the minimum tolerated quality. An illustration of the optimization process is shown in Fig. 3. The optimization problem can be formulated in a global way, where all nodes are displaced simultaneously, or in a local way, where nodes are displaced one-by-one in serie, as done by Garimella, Shashkov, and Knupp (2004).

Other mesh optimization approaches can also be explored, such as those based on a mechanical analogy (Daldoul, 2017). For example, to refine the discretization in a particular region of the domain, a force field can be applied to move the internal nodes towards the area of interest, as shown in Fig. 4. The displacement can be determined by linear elasticity PDEs, which can be efficiently solved in distributed memory architectures. This idea can be also used for improving mesh quality when imposing, as a Dirichlet condition, the displacement of nodes towards the interface.

The work can be carried out in Python or Matlab to take advantage of their constrained optimization algorithms, however, the student is highly encouraged to implement her/his own optimization algorithm to carry out the task. It is not required to be familiar with C++ nor MPI, as there is no obligation at this time to analyze algorithms in a distributed memory architecture.

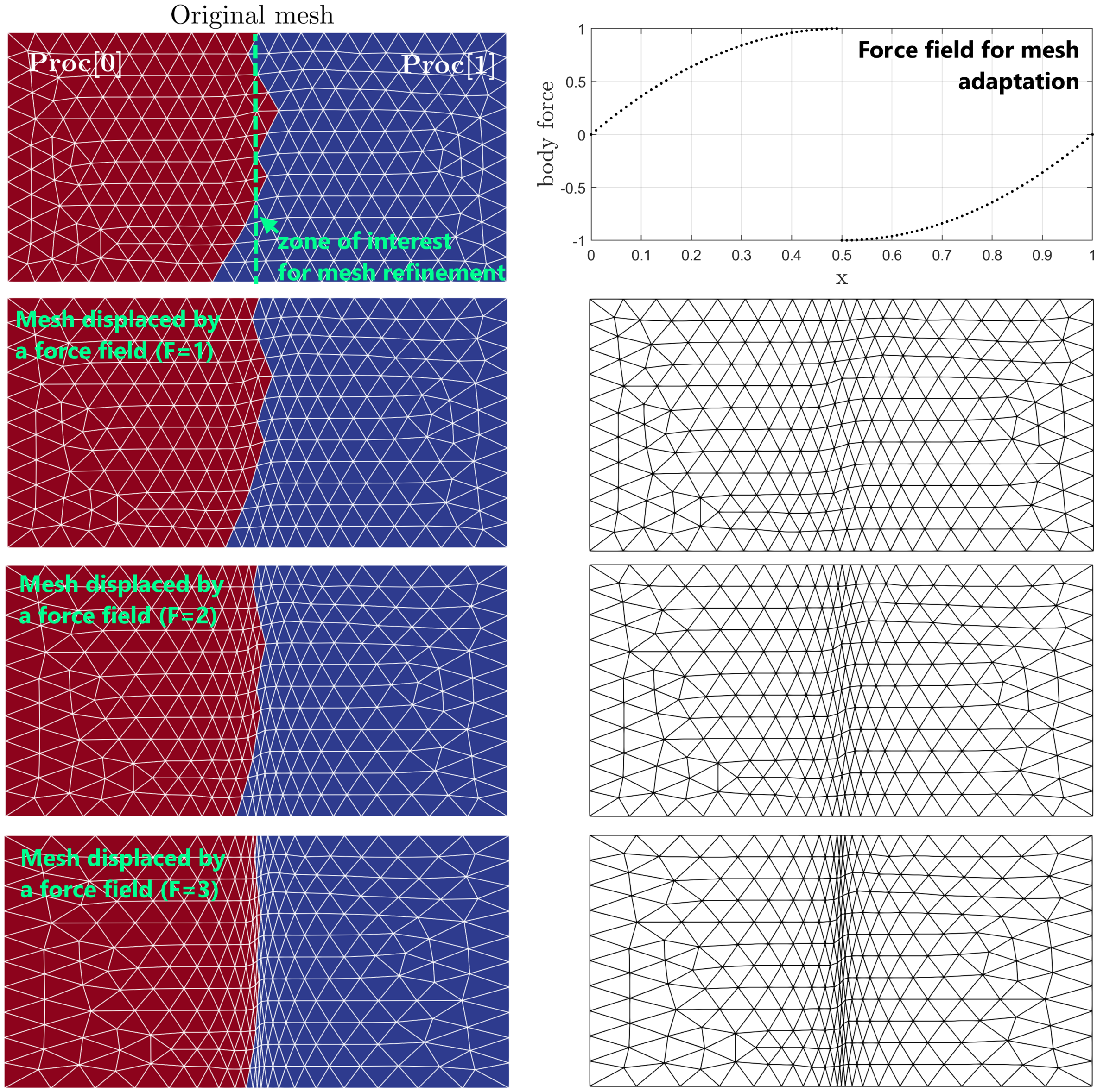

Figure 4: *Mesh distributed in 2 processes with an initial discretization of uniform element size. A body force field is applied in the domain to move the nodes towards the zone of interest. Slip boundary condition is used on the boundaries. The example is obtained using our own code (C++, MPI, PETSc).*

References

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